NEW YORK UNIVERSITY
INSTITUTE OF MATHEMATICAL SCIENCES
LIBRARY
25 Waverly Place, New York 3, N. Y.

IMM-NYU 169

THE DOCK PROBLEM

By

K. O. FRIEDRICHS and HANS LEWY

IMM-169
c. 1

Prepared under
Navy Contract N6ori-201 Task Order No. 1
By the
Institute for Mathematics and Mechanics
New York University

October, 1947

The Dock Problem

by

K. O. Friedrichs and Hans Lewy

Suppose one half-plane of an infinite water surface is covered with a rigid plate, the "dock." How does this dock influence waves standing or traveling perpendicularly to the edge of the dock on the free water surface? This problem may be considered as a special case of the problem of waves on a sloping beach, viz. for a beach with a slope angle of 180°, see the introductory report [4]. For beach angles which are an integer fraction of a right angle, $\omega = \frac{\pi}{2q}$, the problem was solved by Miche [1], Stoker, and Lewy [2,3]; for beach angles of the form $\omega = \frac{p\pi}{2q}$ with relatively prime integers p and q, the problem was solved by Lewy in case p is odd. The present problem corresponds to the case $q = 1$, $p = 2$. Although it may be possible to make $\frac{p}{2q}$ tend to the limit 1 and follow through this passage to the limit in the explicit expressions for the waves, we attack the problem differently, using a method related to the Laplace transformation. It turns out that the problem permits a simple explicit solution which offers the possibility of discussing the nature of the solution and of determining it numerically. We shall in particular discuss the behavior of the wave motion near the edge of the dock, i.e. near the line along which the water surface and the dock meet. In addition, we shall show graphically the amplitudes of the waves on the free surface and of the pressure under the dock.

Aside from its interest as a flow problem, the dock problem is of significance as a new example of a potential problem with different linear boundary conditions on different parts of the boundary.*

Since we are interested in two-dimensional flow, all quantities are assumed constant along any parallel to the edge. We place an (x,y)-coordinate system such that the water fills the lower half-plane $y < 0$, the free water surface is the positive x-axis, $y = 0$, $x > 0$, and the dock is the negative x-axis, $y = 0$, $x < 0$. We set $x + iy = z = r\,e^{i\theta}$. Further we choose the units of length and time such that the frequency of the oscillations of the water surface and the acceleration of gravity have the value one. The water flow is then described by a complex potential function

$$\chi(z)\cos t$$

from which the velocity components u and v are derived:

$$u - iv = \frac{d\chi(z)}{dz}\cos t\ .$$

The boundary condition that the pressure be constant on the free surface leads, by the well known process of linearization, see e.g. Stoker [3,p.6], to the condition

$$\text{I} \qquad \operatorname{Im}\left(\frac{d}{dz}\chi + i\chi\right) = 0, \quad y = 0,\ x > 0\ ,$$

while the condition that the water slides under the dock surface leads to the condition

---

*A solution of the problem of two- and three-dimensional waves over a channel of finite depth with a dock has been announced by A. E. Heins [5].

$$\operatorname{Im}\left(\frac{d}{dz}\chi\right) = 0 , \quad y = 0,\ x < 0 . \tag{II}$$

At infinity, i.e. as $|z| \to \infty$, we further require

$$\chi(z) \sim \chi_{\infty} e^{-iz} , \tag{III}$$

i.e.

$$\chi(z)e^{iz} \to \chi_{\infty} , \quad \theta_{*} \leq \theta \leq 0 ,$$

$$\chi(z) \to 0 , \quad -\pi \leq \theta \leq \theta_{**} ,$$

for any angles θ_{*}, θ_{**} with $-\pi < \theta_{*} < \theta_{**} < 0$ and with an appropriate complex constant $\chi_{\infty} \neq 0$.

These two conditions, which are evidently not contradictory, assure that, at large distances from the dock, the water executes sinusoidal motions near the surface and quiets down in other directions.

At the edge we permit a logarithmic singularity, i.e. we require that, with an appropriate constant k,

$$\chi(z) - k \log z \quad \text{is bounded at} \quad z = 0 . \tag{IV}$$

For the significance of this logarithmic singularity see Stoker [3, p. 11].

We shall show two solutions of the problem: one, $\chi^{R}(z)$, bounded at the edge $z = 0$, the other, given by

$$\chi^{S}(z) = -\frac{d}{dz}\chi^{R}(z) \tag{1}$$

and possessing a logarithmic singularity at the edge. Specifically, the behavior of the solutions at $z = \infty$ is given by

(2) $\sqrt{2}\,\chi^R(z) - e^{\frac{i\pi}{8}} e^{-iz} \to 0, \sqrt{2}\,\chi^S(z) - ie^{\frac{i\pi}{8}} e^{-iz} \to 0, \; z \to \infty,$

if z approaches infinity within a sector which excludes the negative real axis, $-\pi < \theta_* \leq \theta \leq 0$, while if z approaches infinity in a sector which excludes the positive real axis, $-\pi \leq \theta \leq \theta_{**} < 0$,

(3) $$\chi^R(z) \to 0, \quad \chi^S(z) \to 0, \quad z \to \infty.$$

Both statements, (2) and (3), are in agreement in the overlapping parts of the sectors. The behavior at the origin is given by

(4) $$\chi^R(z) \to \frac{1}{2} \quad \text{as} \quad z \to 0,$$

(5) $$\chi^S(z) - \frac{1}{2\pi} \log(-z) \to -\frac{1-C}{2\pi} \quad \text{as} \quad z \to 0,$$

C being the Euler constant.

The relation

(6) $$\chi^R(z)e^{iz}\Big|^{\infty} \Big/ \chi^R(0) = \sqrt{2}\; e^{i\frac{\pi}{8}}$$

is particularly remarkable, since it is in agreement with the value $\eta\sqrt{p/q}$ found for the same ratio for beaches of slope angle $p\pi/2q$, see [2,p. 744], η being an appropriate number of absolute value one. Note that in our case, $p = 2$, $q = 1$, p is not odd.

The solutions $\chi^R(z)$, $\chi^S(z)$ are given by the integrals

$$(7) \qquad \chi^{R}(z) = \frac{1}{2\pi} \int_{\mathcal{P}} e^{\zeta z} \, \psi(\zeta) \, d\zeta \quad ,$$

$$(8) \qquad \chi^{S}(z) = - \frac{1}{2\pi} \int_{\mathcal{P}} e^{\zeta z} \, \zeta \psi(\zeta) \, d\zeta \quad ,$$

in which $\mathcal{P}$ is any path running in the lower half-plane $\eta \leq 0$ from $\zeta = 0$ to $\zeta = \infty$ entering $\zeta = \infty$ in a direction along which ζz has a negative real part and skirting the point $\zeta = -i$ so as to leave it to the right. Such a path $\mathcal{P}$ will then be called "admissible" for the value z.

The function $\psi(\zeta)$ is defined in the lower half-plane, $\eta \leq 0$, $(\zeta \neq 0)$, except for the point $\zeta = -i$, at which it has a pole. It is given by the two equivalent representations

$$(9)_0 \qquad \psi(\zeta) = \zeta^{-\frac{1}{2}}(1 - i\zeta)^{-1}(1 + i\zeta)^{-\frac{1}{2}} \exp \frac{1}{\pi} \int_0^{\zeta} \log \, (i\zeta_*)^{-1} \frac{d\zeta_*}{\zeta_*^2 + 1} \, ,$$

$$(9)_\infty \qquad \psi(\zeta) = \zeta^{-2}(1 + i\zeta^{-1})^{-1}(1 - i\zeta^{-1})^{-\frac{1}{2}} \exp \frac{1}{\pi} \int_{\zeta}^{-i\infty} \log \, (i\zeta_*) \frac{d\zeta_*}{\zeta_*^2 + 1} \, .$$

Note that the function $\int_{\zeta}^{-i\infty} \log \, (i\zeta_*) \frac{d\zeta_*}{\zeta_*^2 + 1}$ is regular in the lower half-plane. The equivalence of the two representations follows from the relation

$$(10) \quad \int_0^{-i\infty} \log\,(i\zeta_*)\frac{d\zeta_*}{\zeta_*^2+1} = \int_0^{\infty} \log\,(i\xi_*)\frac{d\xi_*}{\xi_*^2+1}$$

$$= \frac{i\pi}{2}\int_0^{\infty} \frac{d\xi_*}{\xi_*^2+1} = \frac{i\pi^2}{4}\,.$$

From the relation

$$\int_0^{-1} \log\,(i\zeta_*)\frac{d\zeta_*}{\zeta_*^2+1} = \int_{-i}^{-i\infty} \log\,(\tau i)\frac{d\tau}{\tau^2+1}$$

obtained by substituting $-\frac{1}{\tau}$ for ζ_* and from (10), we conclude

$$(11) \qquad \int_0^{-i} \log\,(i\zeta_*)\frac{d\zeta_*}{\zeta_*^2+1} = \frac{i\pi^2}{8}$$

and we see that $(\zeta + i)\ \psi(\zeta)$ is regular at $\zeta = -i$ and assumes there the value

$$(12) \qquad (\zeta + i)\ \psi(\zeta)\Big|^{-i} = i\ e^{\frac{i\pi}{8}}\,.$$

The path of integration $\mathcal{P}$ is any curve coming from the origin, skirting the point $\zeta = -i$ leaving it to the right, and then going to infinity in a direction in which the real part of ζz is negative. Thus from the expressions $(9)_0$ and $(9)_\infty$ it is clear the $\psi(\zeta)$ behaves at

$\zeta = 0$ and at $\zeta = \infty$ in such a way that the two integrals (7) and (8) represent analytic functions of z which are regular in the lower half-plane, $y \leq 0$, $(z \neq 0)$.

In order to verify that the boundary conditions I and II are satisfied we use different representations of the function $\psi(\zeta)$. From the relations

$$(13)_0 \quad i\int_0^{\zeta} \frac{d\zeta_*}{\zeta_*^2+1} = \frac{1}{2}\log\frac{1+i\zeta}{1-i\zeta}, \qquad \text{except for } \xi = 0,\ |\zeta| \geq 1,$$

$$(13)_\infty \quad i\int_{\zeta}^{\infty} \frac{d\zeta_*}{\zeta_*^2+1} = \frac{1}{2}\log\frac{1+i\zeta^{-1}}{1-i\zeta^{-1}}, \quad \text{except for } \xi = 0,\ |\zeta| \leq 1,$$

we find

$$(14)_0 \quad \psi(\zeta) = \zeta^{-\frac{1}{2}}(1+\zeta^2)^{-\frac{3}{4}} \exp\frac{1}{\pi}\int_0^{\zeta} \log\zeta_*^{-1}\frac{d\zeta_*}{\zeta_*^2+1},$$

$$(15)_0 \quad \psi(\zeta) = \zeta^{-\frac{1}{2}}(1-i\zeta)^{-1}(1+\zeta^2)^{-\frac{1}{4}} \exp\frac{1}{\pi}\int_0^{\zeta} \log(-\zeta_*)^{-1}\frac{d\zeta_*}{\zeta_*^2+1},$$

valid for $\eta \leq 0$ except for $\xi = 0$, $\eta \leq -1$,

$$(14)_{\infty} \quad \psi(\zeta) = \zeta^{-2}(1+\zeta^{-2})^{-\frac{3}{4}} \exp\frac{1}{\pi}\int_{\zeta}^{\infty} \log\zeta_*\frac{d\zeta_*}{\zeta_*^2+1},$$

valid for $\eta \leq 0$ except for $\xi = 0$, $\eta \geq -1$. These representations make it evident that:

$\psi(\zeta)$ *is real positive on the positive real axis,*
$\zeta = \xi = 0, \quad \eta = 0,$

$(\zeta + i) \; \psi(\zeta)$ *is real positive on the negative*
real axis, $\zeta = \xi < 0, \eta = 0.$

For values $z = x < 0$, $y = 0$, on the negative real z-axis we may take as the path $\mathcal{P}$ the positive real ζ-axis, $\zeta = \xi > 0$, $\eta = 0$. From the representations (7) and (8) we can then see that $\chi^R(z)$ and hence also $\frac{d}{dz} \chi^R(z)$ and $\frac{d}{dz} \chi^S(z) = -\frac{d^2}{dz^2} \chi^R(z)$ are real for $z = x < 0$, $y = 0$. The boundary condition II is therefore satisfied. In order to *verify that the boundary condition I is satisfied* we observe that the integrand in

$$\left(\frac{d}{dz} + i\right) \chi^R(z) = \int_{\mathcal{P}} e^{\zeta z} (\zeta + i) \;\; \psi(\zeta) \, d\zeta$$

is regular at $\zeta = -i$. Hence we can deform the path $\mathcal{P}$ into the negative real ζ-axis, $\zeta = \xi < 0$, $\eta = 0$, if z lies on the positive real z-axis, $z = x > 0$, $y = 0$. Since $(\zeta + i) \; \psi(\zeta)$ is positive real for $\zeta = \xi < 0$, $\eta = 0$, we see that $\left(\frac{d}{dz} + i\right) \chi^R(z)$ is real for $z = x > 0$, $y = 0$. The same statement follows similarly for $\chi^S(z)$. The boundary condition I is therefore satisfied.

To derive *the behavior* of χ^R and χ^S *at infinity* we make use of the pole of $\psi(\zeta)$ at $\zeta = -i$. Suppose the point $z = re^{i\theta}$ lies in any sector excluding the positive real axis, $-\pi \leq \theta \leq \theta_{**} < 0$. Then ζz has a negative real part on any ray $\zeta = \rho e^{i\lambda}$, λ = constant whose angle λ lies in the interval $-\frac{\pi}{2} < \lambda < -\frac{\pi}{2} - \theta_{**}$. Since any such ray in the lower half-plane, run through from 0 to ∞, leaves the point $\zeta = -i$ to the right, it is admitted as a path $\mathcal{P}$. If we now let z approach ∞ it is clear that only the neighborhood of $\zeta = 0$ will contribute asymptotically to the integrals (7) and (8). By $(14)_0$ the function $\psi(\zeta)$ behaves like $\zeta^{-\frac{1}{2}}$ at the origin; consequently, $\chi^R(z)$ behaves like $\frac{1}{2\sqrt{-\pi z}}$ for large values of z and hence vanishes as $z \to \infty$. If, on the other hand, $z = re^{i\theta}$ lies in the sector $-\pi \leq \theta_* \leq \theta \leq 0$, we shift our path to any ray $\zeta = \rho e^{i\lambda}$, $\lambda =$ constant, with $-\frac{3}{2}\pi - \theta_* < \lambda < -\frac{\pi}{2}$, and then argue as before. In doing so, however, we cross the pole at $\zeta = -i$. The residue contributed by the pole is just $\frac{1}{\sqrt{2}} e^{\frac{i\pi}{8}} e^{-iz}$ for $\chi^R(z)$ and $\frac{1}{\sqrt{2}} ie^{\frac{i\pi}{8}} e^{-iz}$ for $\chi^S(z)$ as seen from (12). Thus the formulas (2) and (3) are justified.

To determine the behavior of the functions $\chi(z)$ at the edge $z = 0$ we must establish an expansion of $\psi(\zeta)$ for large values of ζ. Since

$$(16) \quad \int_{\zeta}^{\infty} \log \zeta_* \frac{d\zeta_*}{\zeta_*^2+1} = \zeta^{-1}(\log \zeta + 1) - \frac{1}{3}\zeta^{-3}(\log \zeta + \tfrac{1}{3}) + \dots$$

we have from $(14)_\infty$

$$(17) \quad \psi(\zeta) = \zeta^{-2} + \frac{1}{\pi}\zeta^{-3}(\log \zeta + 1) + 0(\zeta^{-4} \log^2 \zeta).$$

Thus $\psi(\zeta)$ vanishes at infinity like ζ^{-2} and the integral

$$(18) \qquad P = \frac{1}{2\pi} \int_{\mathcal{P}} \psi(\zeta) d\zeta$$

is absolutely integrable and does not depend on which path $\mathcal{P}$ is chosen. Hence, $\chi^R(z)$ approaches P if z approaches 0 through values for which the path $\mathcal{P}$ is admissible, see page 5. Consequently, if the function $\chi^R(z)$ is defined for z = 0 by $\chi^R(0) = P$, it is continuous there.

To determine the value of P we consider the integral

$$(19) \qquad Q = \frac{1}{2\pi}\int_{\mathcal{P}} \left\{ \zeta\psi(\zeta) - \zeta(\zeta^2+1)^{-1} \right\} d\zeta \ ,$$

which is also independent of the choice of $\mathcal{P}$. The term subtracted from $\zeta\psi(\zeta)$ is so chosen that the integrand vanishes at infinity like $\frac{1}{\pi}\zeta^{-2}\log\zeta$, as seen from (17), and remains integrable at $\zeta = 0$.

If one takes the path $\mathcal{P}$ in (18) and (19) as the positive ξ-axis, it becomes clear that both P and Q are real.

We now combine the two integrals to

$$(20) \qquad Q + iP = \frac{1}{2\pi}\int_{\mathcal{P}} \left\{ (\zeta+i)\ \psi(\zeta) - \zeta(\zeta^2+1)^{-1} \right\} d\zeta \ .$$

Since $(\zeta+i)\ \psi(\zeta)$ is regular in the whole lower half-plane $(\zeta \neq 0)$, we may swing the path of integration into the negative ξ-axis, taking the residue due to the second contribution into account. We obtain

$$(21) \qquad Q + iP = \frac{1}{2\pi}\int_0^{-\infty} \left\{ (\zeta+i)\ \psi(\zeta) - \zeta(\zeta^2+1)^{-1} \right\} d\zeta + \frac{i}{2} \ .$$

Now, $(\zeta + i)\ \psi(\zeta)$ is real on the negative ξ-axis. Hence we have

$$(22) \qquad P = \frac{1}{2} ,$$

$$(23) \qquad Q = \frac{1}{2\pi} \int_0^{-\infty} \left\{ (\zeta + i)\, \psi(\zeta) - \zeta (\zeta^2 + 1)^{-1} \right\} d\zeta .$$

Relation (22) yields formula (4) since

$$(24) \qquad \chi^R(z) \rightarrow P \quad \text{as} \quad z \rightarrow 0 ,$$

as was shown above.

We now apply the same process in the opposite direction. We consider the integral

$$(25) \qquad R = \frac{1}{2\pi} \int_0^{-\infty} \left\{ \zeta (\zeta + i)\ \psi(\zeta) - [1 + \frac{1}{\pi}\ \zeta (\zeta^2 + 1)^{-1} (\log \zeta + i\pi + 1)] \right\} d\zeta$$

which, by (17), converges absolutely and permits swinging the path of integration back to the positive real axis, except for a residue due to the second contribution. We note that R is real since $(\zeta + i)\,\psi(\zeta)$ and $\log \zeta + i\pi$ are real on the negative ξ-axis. We now combine R with Q, as given by (25) and (23), to

$$(26)\quad R - iQ = \frac{1}{2\pi}\int_0^{-\infty}\Big\{(\zeta^2+1)\,\psi(\zeta) - \big[1+\tfrac{1}{\pi}\,\zeta(\zeta^2+1)^{-1}(\log\zeta+1)\big]\Big\}\,d\zeta$$

and swing the path back into a path $\mathcal{P}$. Since $(\zeta^2+1)\,\psi(\zeta)$ is regular in the lower half-plane, we obtain

$$(27)\quad R - iQ = \frac{1}{2\pi}\int_{\mathcal{P}}\Big\{(\zeta^2+1)\,\psi(\zeta) - \big[1+\tfrac{1}{\pi}\,\zeta(\zeta^2+1)^{-1}(\log\zeta+1)\big]\Big\}\,d\zeta - \frac{1}{4} - \frac{i}{2\pi}\,.$$

The integral in (27) is real if $\mathcal{P}$ is taken as the positive ξ-axis. Consequently, we have

$$(28)\qquad Q = \frac{1}{2\pi}\,.$$

From this result we can derive formula (5), which gives the behavior of $\chi^S(z)$ at $z = 0$. We first observe that, by the definition (19) of Q, relation (28) implies

$$(29)\qquad \frac{1}{2\pi}\int_{\mathcal{P}} e^{\zeta z}\Big\{\zeta\,\psi(\zeta) - \zeta(\zeta^2+1)^{-1}\Big\}\,d\zeta \to \frac{1}{2\pi}$$

if z approaches 0 through values for which $\mathcal{P}$ is admissible. Thus, by (8), the behavior of $\chi^S(z)$ at $z = 0$ is reduced to that of

$$\int_{\mathcal{P}} e^{\zeta z}\,\zeta(\zeta^2+1)^{-1}\,d\zeta \quad .$$

In order to discuss this integral we may lead the path $\mathcal{P}$ first along the positive ξ-axis from 0 to any point $\zeta = \alpha$, $\alpha > 0$, and then to infinity in such a way that it eventually runs along a straight line on which ζz is real and negative. Consequently, we may write

$$(30)\quad \int_{\mathcal{P}} e^{\zeta z}(\zeta^2+1)^{-1}\zeta d\zeta = \int_0^{\alpha} e^{\zeta z}(\zeta^2+1)^{-1}\zeta d\zeta$$

$$+\int_{-\alpha z}^{\infty} e^{-\sigma}(\sigma^2+z^2)^{-1}\sigma d\sigma$$

$$= \int_0^{\alpha} (e^{\zeta z}-1)(\zeta^2+1)^{-1}\zeta d\zeta$$

$$-\log(-z) + \tfrac{1}{2}(1-e^{-\alpha z})\log(z^2+\alpha^2 z^2)$$

$$+\frac{1}{2}\int_{-\alpha z}^{\infty} e^{-\sigma}\log(\sigma^2+z^2)d\sigma \quad .$$

The path in the last integral, which begins at the point $-\alpha z$ in the upper half-plane, enters infinity along a section of the positive ξ-axis; if $-\alpha z$ lies in the left half-plane the path skirts the point $\sigma = -iz$ so as to leave it to the right. Letting now z approach 0 we obtain

$$(31)\quad \int_{\mathcal{P}} e^{\zeta z}(\zeta^2+1)^{-1}\,\zeta d\zeta + \log(-z) \to \int_0^{\infty} e^{-\sigma}\log\sigma d\sigma = -C,$$

C being the Euler constant. Consequently we have from (8), (29), and (31)

$$(32) \qquad \chi^S(z) - \frac{1}{2\pi}\log(-z) \rightarrow -\frac{1-C}{2\pi}$$

as $z \rightarrow 0$. Thus formula (5) is established.

Since the function $-\chi^S(z)$ is the derivative of $\chi^R(z)$, the formula (5) gives at the same time a refined behavior of $\chi^R(z)$ at $z = 0$,

$$(33) \qquad \chi^R(z) = \frac{1}{2} - \frac{1}{2\pi}\left\{ z\log(-z) - (2-C)z \right\} + O(z),$$

the symbol $O(z)$ representing a function for which $z^{-1}O(z) \rightarrow 0$ as $z \rightarrow 0$.

Refined asymptotic representations of the functions $\chi^R(z)$ and $\chi^S(z)$ at $z = 0$ could be derived by continuing the process of swinging the path of integration of appropriate integrals forward and backward; but this procedure becomes very cumbersome. Two different methods of deriving asymptotic expansions of analytic functions satisfying the two boundary conditions I and II on the two rays of the real axis will be given in a later publication.

Explanation of the figures

The figures show the values of the velocity potential $\varphi(x,0) = \mathrm{Re}\,\chi(x)$ along the dock, $x < 0$ and along the free surface $x > 0$. The elevation of the free surface is given by

$$\eta = g^{-1}\sigma\,\varphi(x,0)\,\sin\sigma t, \quad x > 0,$$

the pressure under the dock surface by

$$p = \rho\sigma\,\varphi(x,0)\,\sin\sigma t, \; x < 0.$$

The unit of length is chosen as $g\sigma^{-2}$.
Figures 1 and 2 refer to the wave motion described by the function $\chi^R(z)$ and $\chi^S(z)$ respectively.

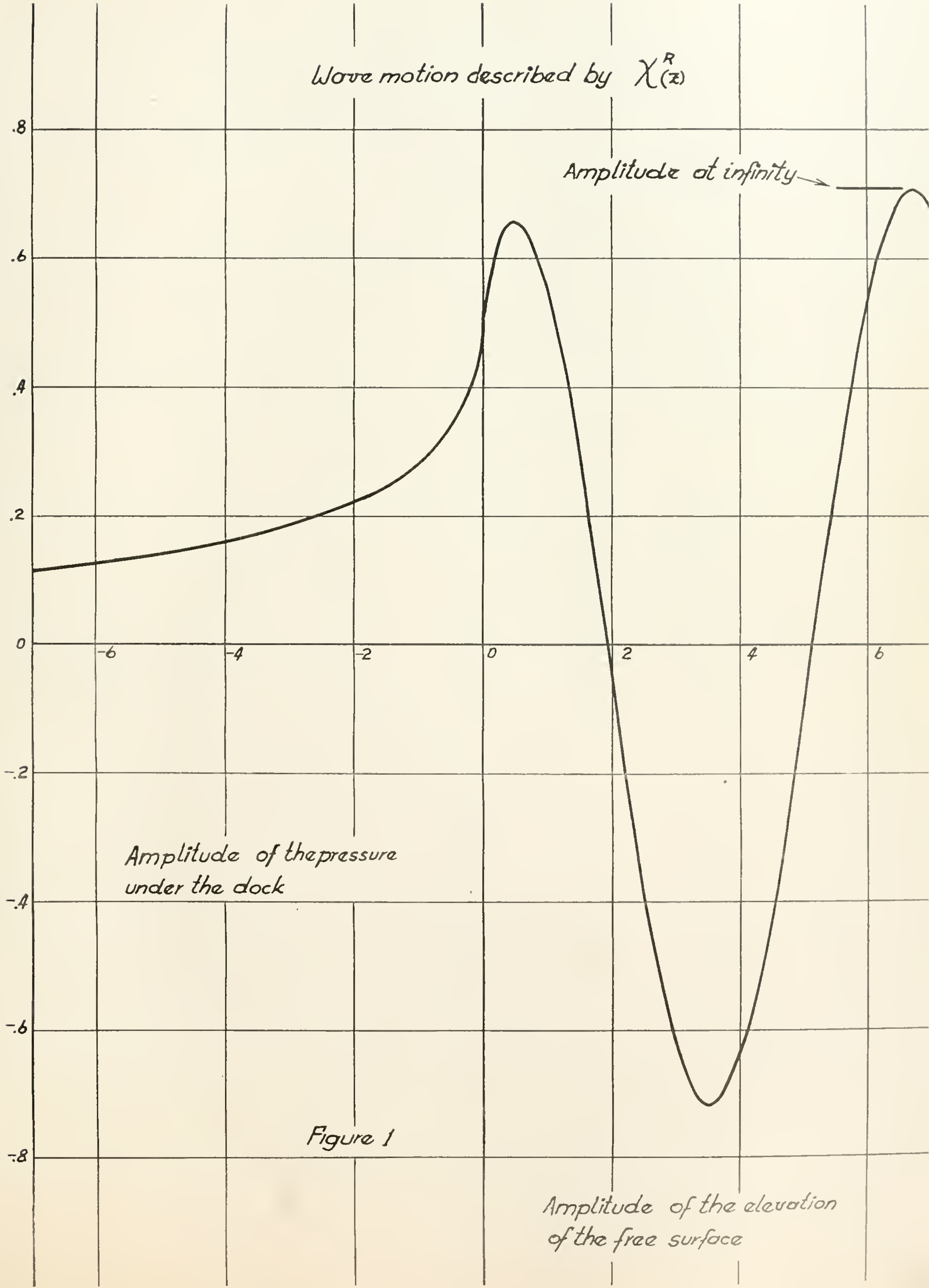
Wave motion described by $\chi^R_{(z)}$
.8
.6
.4
.2
0
-.2
-.4
-.6
-.8
-6
-4
-2
0
2
4
6
Amplitude at infinity
Amplitude of the pressure under the dock
Amplitude of the elevation of the free surface

Figure 1

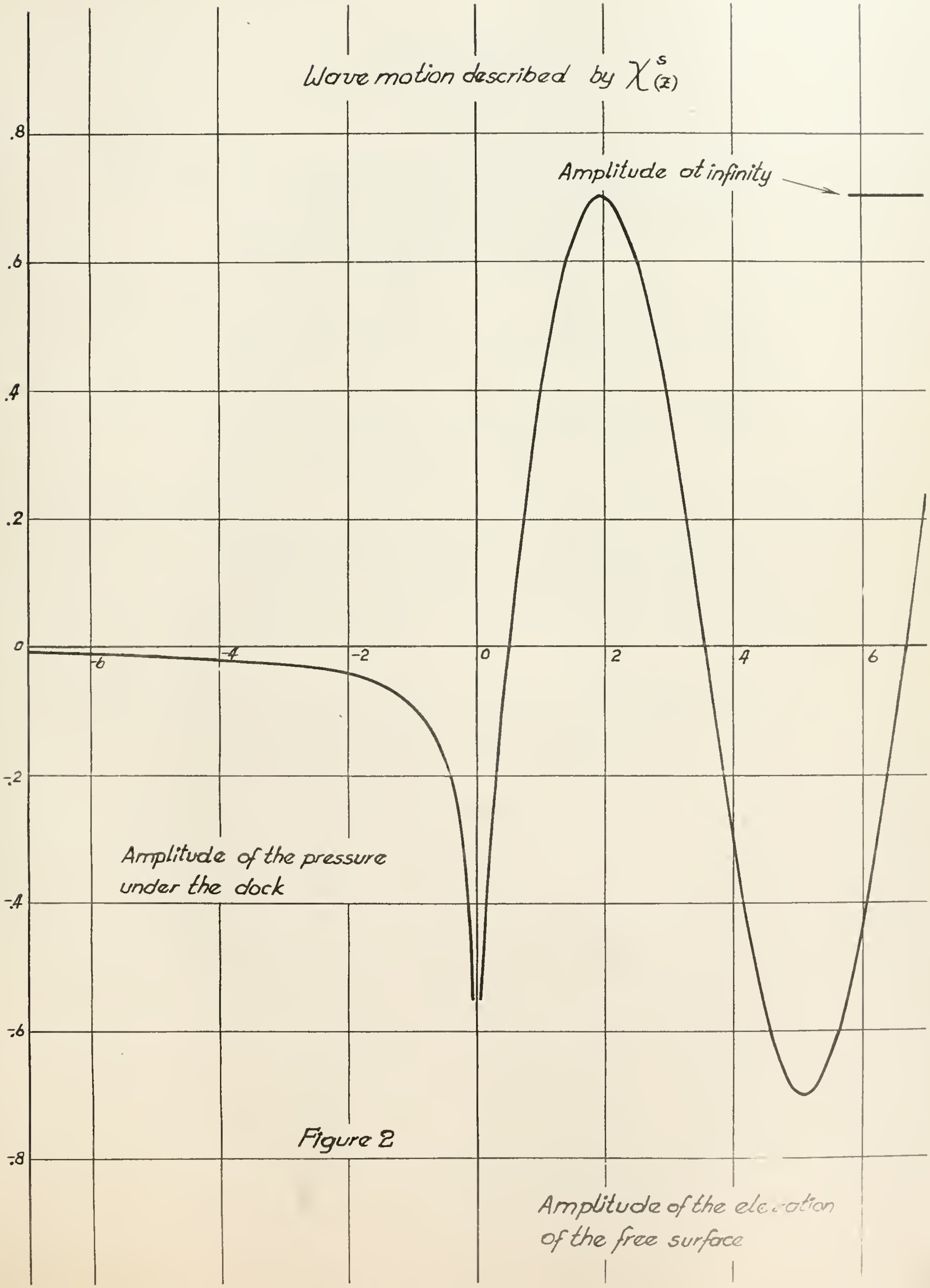
Wave motion described by $\chi^{s}_{(x)}$
.8
.6
.4
.2
0
-.2
-.4
-.6
-.8
-6
-4
-2
0
2
4
6
Amplitude at infinity
Amplitude of the pressure under the dock
Amplitude of the elevation of the free surface

Figure 2

BIBLIOGRAPHY

[1] Miche, A.: Mouvements ondulatoires de la mer en profondeur constante ou décroissante, Ann. des ponts et chaussées, 1944, pp. 25-78, 131-164, 270-292, 369-406.

[2] Lewy, H.: Water Waves on Sloping Beaches, Bull. Am. Math. Soc., Vol. 52, 1946, pp. 737-775.

[3] Stoker, J. J.: Surface Waves in Water of Variable Depth, AMG-NYU No. 153, Quarterly of Appl. Math., Vol. V, No. 1, April 1947.

[4] Stoker, J. J.: Waves over Beaches of Small Slope, Under a Dock or a Cliff, and Past Plane Barriers, Institute for Mathematics and Mechanics-New York University Report No. 161, September, 1947.

[5] Heins, A. E.: Water Waves over a Channel with a Dock, Bull. of the Amer. Math. Society, Vol. 53, Nr. 5, p. 497, 1947.

Date Due

PRINTED IN U. S. A.

NYU c.1
IMM-169

Friedrichs
The docks problem

NYU c.1
IMM-169 Friedrichs
AUTHOR
The docks problem
TITLE

DATE DUE	BORROWER'S NAME	ROOM NUMBER

N. Y. U. Institute of
Mathematical Sciences
25 Waverly Place
New York 3, N. Y.